Q5 review key

Stat 301

Summer 2019

(1) A company that manufactures coffee for use in commercial machines monitors the caffeine content in its coffee. The company randomly selected 50 samples of coffee every hour from its production line and determines the caffeine content. From historical data, the caffeine content is known to have a standard deviation of 7.1 mg. During one 1-hour period, the random sample of 50 had a sample mean of 110 mg.

(a) The caffeine content should usually be 107 mg. Is there sufficient evidence that the mean caffeine content is more than the usual amount?

1. Hypotheses:

$H_0 : \mu = 107$ and $H_a : \mu > 107$ Assumptions: random (yes), independence (yes because random), normal (yes it should be since $n \geq 30$)

2. Test statistic:

$se_{mean} = \frac{7.1}{\sqrt{50}} = 1.0041, \ z = \frac{110-107}{1.004} = 2.99$

3. Rejection Region:

Reject H_0 if $pvalue \leq \alpha$ With $H_a :>$, $pvalue = P(Z > z_{calc})$. $pvalue = P(Z > z_{calc}) = 1 - P(Z < z_{calc}) = 1 - P(Z < 2.99) = 1 - 0.9986 = 0.0014$.

Since $0.0014 \leq \alpha(0.05)$, we will reject H_0.

4. Conclusion (results and conclusion in context):

Since H_0 was rejected, there is sufficient evidence the true average caffeine content is more than the usual amount of 107 mg.

(b) What kind of error could have been made? Define the error and explain it in the context of the scenario.

Since we rejected H_0, the only kind of error we could have made was a Type I error, α. It is defined as $P(Reject\ H_0 | H_0\ true)$, rejecting the null hypothesis when the null hypothesis is true. We think the caffeine content is more than 107 mg but it is not.

(c) Estimate μ, the true average caffeine content of the coffee, with 95% confidence. Interpret.

$z^{\star} = z_{\alpha/2} = z_{0.05/2} = z_{0.025} = 1.96$

$110 \pm (1.96)\left(\frac{7.1}{\sqrt{50}}\right) = 110 \pm 1.968 = 108.03, 111.97$ mg

We are 95% confident the true mean caffeine content is between 108.03 and 111.97 mg.

(d) Suppose that the company would prefer a margin of error (bound) for the next batch to be 1.15. What sample size would be needed to get a bound of 1.15, while maintaining 95% confidence?

$n = \left(\frac{z^{\star}\sigma}{bound}\right)^2 = \left(\frac{(1.96)(7.1)}{1.15}\right)^2 = 146.4310442$ and since it is a sample size, we always round up (regardless of normal rounding conventions). So $n \approx 147$

(2) It is thought that more than 70% of all faults in transmission lines are caused by lightning. In a random sample of 200 faults from a large data base, 151 are due to lightning.

(a) Is there sufficient evidence that the proportion of faults in transmission due to lightning strikes is different from 70%? Conduct a hypothesis test.

1. Hypotheses:

$H_0 : p = 0.7$ $H_a : p \neq 0.7$ $\hat{p} = \frac{X}{n} = \frac{151}{200} = 0.755$

Assumptions: random (yes), independence (yes because random), $np_0 = 200(0.7) = 140 \geq 10$ and $nq_0 = 200(0.3) = 60 \geq 10$, also $n \geq 60 = 200$

2. Test Statistic:

$se_{p_0} = \sqrt{\frac{p_0 q_0}{n}} = \sqrt{\frac{.7 * .3}{200}} = 0.0324$

$z = \frac{\hat{p}-p_0}{se_{p_0}} = \frac{0.755-0.70}{0.0324} = 1.7$

3. Rejection Region:

Reject H_0 if $pvalue \leq \alpha$. Since $H_a :\neq$, $pvalue = 2P(Z > z_{calc}) = 2[1 - P(Z < 1.7)] = 2(0.0446) = 0.0891$. Since $0.0891 \nleq \alpha(0.05)$, we will fail to reject H_0.

4. Conclusion (results and conclusion in context): Since we did not reject H_0, the proportion of faults in transmission due to lightning strikes is still right around 70% (it is not different from 70%).

(b) What kind of error could have been made? Define the error and explain it in the context of the scenario.

Since we did not reject H_0, the only kind of error we could have made was a Type II error (β). It is defined as $P(Fail\ to\ Reject\ H_0|H_0\ false)$, not rejecting a false hypothesis. We think the faults in transmission due to lightning is not different from the ususal 70%, when it would be different from the 70%.

(c) Estimate p, the true proportion of faults in transmission due to lightning strikes, with 95% confidence. Interpret.

$z^\star = z_{\alpha/2} = z_{0.05/2} = z_{0.025} = 1.96$

$\hat{p} \pm z^\star\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.755 \pm (1.96)\sqrt{\frac{(0.755)(0.245)}{200}} = 0.755 \pm (1.96)(0.0304) = 0.755 \pm 0.0596 = (0.6954, 0.8146)$. We are 95% confident the true proportion of faults of transmission due to lightning strikes is between 69.54% and 81.46%.

(d) Suppose that next sample should have a bound of 3%. What sample size would be needed to get a bound of 3%, while maintaining 95% confidence?

$n = \hat{p}\hat{q}\left(\frac{z^\star}{bound}\right)^2 = (0.755)(0.245)\left(\frac{1.96}{0.03}\right)^2 = 789.5555111 \Rightarrow n \approx 790$

(3) In 1882 Michelson measured the speed of light (usually denoted as c in Einstein's equation $E = mc^2$). His values are in km/sec and have 29000 subtracted from them for adjustment in ease of analyses. He reported the results of 23 random trials with a mean of 756.22 (km/sec) and standard deviation of 107.12 (km/sec).

(a) Suppose previous experiments of Michelson found that the speed of light was 750 (after subtracting 29,000 from it just like in the previous experiment). Is there sufficient evidence that the speed of light is significantly different from the previous result of 750? Let $\alpha = 0.02$.

1. Hypotheses:

$H_0 : \mu = 750$ and $H_a : \mu \neq 750$ Assumptions: random (yes), independence (yes because random), normal (it should be but hard to tell but we will assume it is)

2. Test statistic:

$se_{mean} = \frac{107.12}{\sqrt{23}} = 22.34$, $t = \frac{756.22-750}{22.34} = 0.278$

3. Rejection Region:

Since $H_a :\neq$, we can reject H_0 if $|t_{calc}| \geq |t_{\alpha/2}|$. $df = n-1 = 23-1 = 22$ $t_{\alpha/2,df} = t_{0.02/2,22} = 2.508$. Since $|0.278| \ngeq |2.508|$, we will fail to reject H_0.

4. Conclusion (results and conclusion in context):

Since H_0 was not rejected, there is not sufficient evidence the true average speed of light is different than previous experiments' results of 750 km/sec.

(b) What kind of error could have been made? Define the error and explain it in the context of the scenario.

Since we did not reject H_0, the only kind of error we could have made was a Type II error (β). It is defined as $P(Fail\ to\ Reject\ H_0|H_0\ false)$, not rejecting a false hypothesis. We think the speed of light is 750 but it is different than previous results.

(c) Estimate μ, the true speed of light with 98% confidence. Interpret.

$t^\star = t_{\alpha/2,df=n-1} = t_{0.02/2,22} = t_{0.01,22} = 2.508$

$756.22 \pm (2.508)\left(\frac{107.12}{\sqrt{23}}\right) = 756.22 \pm 56.0188486 = 700.2, 812.2$ km/sec.

We are 98% confident the true speed of light is between 700.2 and 812.2 km/sec.

(4) Researchers speculate that drivers who do not wear a seatbelt are more likely to speed than drivers who do wear one. A random sample of 40 drivers was taken. In the experiment, the people were clocked to see how fast they were driving (mph) and then were stopped to see whether or not they were wearing a seatbelt. The following table is the summarized data:

Seatbelt?	mean	sd	n
No	72.5	8.816	20
Yes	65.33	7.487	20

(a) Is there sufficient evidence that the average speed for non-seatbelt wearers differs from those drivers that do wear a seatbelt? Let $\alpha = 0.10$

$$H_0 : \mu_1 = \mu_2 \text{ vs. } H_a : \mu_1 \neq \mu_2$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{72.5 - 65.33}{\sqrt{\frac{8.816^2}{20} + \frac{7.487^2}{20}}} = 2.771$$

We can reject H_0 if $|t_{calc}| \geq t_{\alpha/2,df}$ where $df = min(n_1 - 1, n_2 - 1) = min(20-1, 20-1) = 19$ and we assume $\alpha = 0.10$ so $t_{\alpha/2,df} = t_{0.05,19} = 1.729$. Since $|2.771| \geq 1.729$, H_0 is rejected. There is a significant difference in the speeds of drivers who do not wear seatbelts as compared to those who wear seatbelts.

(b) Estimate the true difference in means of the speeds of drivers who do not wear seatbelts as compared to those who wear seatbelts with 90% confidence and interpret.

$$t^\star = t_{\alpha/2,df} = t_{0.05,19} = 1.729$$

$$\bar{x}_1 - \bar{x}_2 \pm t^\star \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 72.5 - 65.33 \pm (1.729)\sqrt{\frac{8.816^2}{20} + \frac{7.487^2}{20}}$$
$$= 7.17 \pm (1.729)(2.586) = 7.17 \pm 4.47 = (2.7, 11.64)$$

We are 90% confident the true difference in means of the speeds of drivers who do not wear seatbelts as compared to those who wear seatbelts is between 2.7 and 11.64 mph.

(c) State the kind of error that could have been made. **DESCRIBE IT IN CONTEXT OF THE DATA**
Since H_0 was rejected, a Type I error (α) could have been made, thinking that there is a difference in the speed of drivers when there isn't.

(5) Many freeways have service (or logo) signs that give information on attractions, camping, lodging, food, and gas services prior to off-ramps. An article reported that in one investigation, six sites along Virginia interstate highways where service signs are posted were selected randomly. For each site, crash data was obtained for a three-year period before distance information was added to the service signs for a one-year period afterward. The summary statistics for the data of the number of crashes per year before and after the sign changes were made, are as follows:

variable	n	mean	sd
before	6	58	35.32
after	6	52.17	28.57
difference	6	-5.83	19.69

(a) Is there sufficient evidence that there is a decrease in accidents after the signs added distance information?

In this case, since differences are calculated as $after - before$ and we want to know if there is a decrease *after*, the hypothesized difference should be < 0. It could have been done the other way around and all it would do is make $\overline{X}_d$ positive, s_d would not change, the t score would be positive, the rejection region would change to the upper tail, and the result would not change.

$$H_0 : \mu_D = 0 \text{ vs. } H_a : \mu_D < 0$$

$$t = \frac{\overline{X}_d}{s_d/\sqrt{n}} = \frac{-5.83}{19.69/\sqrt{6}} = -0.726$$

H_0 can be rejected if $t_{calc} \leq t_{\alpha,df}$ where $df = n - 1 = 6 - 1 = 5$ and we assume $\alpha = 0.05$. One thing to make note of here is that the t value we look on the table for using *alpha* should be negative. The reason is because although the t table has only positive values, it is only for right-tail areas. The curve is symmetric just like the Z table so any value for left tail areas (like ours with $H_a :<$) need to be negative rather than positive. To find t, $t_{\alpha,df} = t_{0.05,5} = -2.015$. Since $-0.726 \nleq -2.015$, we fail to reject H_0. There is no significant difference in accidents before and after the signage, or the signage did not seem to be effective.
(b) Estimate the true mean difference in accidents before and after the signage change with 99% confidence and interpret.

First, find $t^\star$.

$$t^\star = t_{\alpha/2,df} = t_{0.01/2,5} = t_{0.005,5} = 4.032$$

$$\begin{aligned}\overline{X}_d \pm t^\star\left(s_d/\sqrt{n}\right) &= -5.83 \pm (4.032)\left(19.69/\sqrt{6}\right) = -5.83 \pm (4.032)(8.038)\\ &= -5.83 \pm 32.41 = (-38.24, 26.58)\end{aligned}$$

We are 99% confident the true mean difference of accidents before and after the signage change is between -38 and 27. But logically this does not make sense. All this tells us is that there is no difference in accidents (just like the hypothesis test in part a because if the hypothesized value is in the CI ($\mu_d = 0$), then you cannot reject H_0).
(c) State the kind of error that could have been made. **DESCRIBE IT IN CONTEXT OF THE DATA**
With H_0 not rejected, a Type II error (β) could have been made, thinking that there is no change in accidents but there could be a decrease in the accidents.